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EFFECTS OF GRAVITATIONAL BACKREACTION ON SMALL-SCALE STRUCTURE OF COSMIC STRINGS

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ABSTRACT

We show that gravitational backreaction limits the growth of small-scale structure on cosmic strings. A "two-scale" scaling solution results, with small-scale structure and loop formation scaling as $\Gamma_k G\mu$ times the horizon, where $\Gamma_k \sim 50$. There are no more than $(\Gamma_k G\mu)^{-1}$ kinks per horizon length of long string. Gravitational radiation from the long strings is dominated by emission from small scales and is similar, both in frequency and amplitude, to that from loops chopped off the network.

Recently, numerical simulations [1] of cosmic string networks [2] have focused on the evolution of scales that are small relative to the correlation length ξ of the long strings. The long string evolution is described quite well by the "one-scale" scaling solution [2], with ξ growing as the horizon size $2t$. However, in some of the simulations [3], neither the size of small loops chopped off the network, nor the small-scale structure on the long strings, appears to be scaling with horizon size. There is lively debate [4] on the amount, generation, and scale of structure on the long strings. The characteristic size of the daughter loops is also uncertain, and must be known in order to compute the amount of gravitational radiation emitted by the loops [5]. This in turn gives an upper bound to the string parameter $G\mu$ — crucial to the cosmic string galaxy formation scenario [6] — which can be obtained from millisecond-pulsar constraints [7] on the gravitational wave background.

In this Letter, we show that the gravitational backreaction [8] of the radiating strings limits the growth of small-scale structure. A "two-scale" scaling solution results, with small-scale structure and loop formation scaling as $\Gamma_k G\mu$ times the horizon, where $\Gamma_k \sim 50$ is of order the gravitational power coefficient Γ of closed loops [9]. This sets the number of kinks [10], which are discontinuities in the tangent vector of the long strings. With $G\mu \sim 10^{-6}$, there are approximately 10^4 kinks per horizon length of long string.

Gravitational radiation is emitted because of the interaction between left-moving and right-moving kinks. Consider a long string of proper length L, carrying N left-moving and N right-moving kinks. There are, on average, n^2 kink collisions per unit proper length per unit time, where $n \equiv N/L$ is the linear kink density. We define $l \equiv n^{-1}$ as the typical inter-kink distance. During each collision, the transverse shape of the string changes, with a quadrupole moment of order μl^3 changing in a time of order l. We can use the quadrupole formula [11] to estimate the amount of energy ΔE emitted in gravitational waves after each encounter : $\Delta E \sim (G\mu)\ \mu l$. Dimensional considerations prevent higher multipoles from changing the functional dependence of ΔE on l, so we introduce a dimensionless constant of proportionality C_k : $\Delta E = C_k G\mu\, f(\theta_1, \theta_2, \phi)\mu l$, where

$f(\theta_1, \theta_2, \phi)$ is a geometrical factor of order unity depending on the opening angles θ_1, θ_2, and the relative inclination ϕ of the kinks. Note that we have put all velocity dependence, i.e., relativistic corrections, in C_k. The radiative power per unit proper length of long string is thus

$$\frac{d^2E}{dt\,dL} = n^2\, l\ C_k \bar{f} G\mu^2 \equiv n\ \Gamma_k G\mu^2 \quad , \tag{1}$$

where $\bar{f}$ is an average of $f(\theta_1, \theta_2, \phi)$ over kink angles on the string, and $\Gamma_k \equiv C_k \bar{f}$. We find that the gravitational power of a kinky string grows in proportion to the number of kinks that it carries, so that it is n times larger than that from a smooth string of the same length.

We have checked that equation (1) is correct by computing the gravitational power of a closed loop of unit proper length, carrying n left-moving and n right-moving kinks. Using the numerical formalism of Ref. 8 , we find that $n\Gamma_k \sim$ 600, 1500, and 2700, for n = 8, 16, and 32, respectively. These are typical values for kinks with an opening angle of 135 degrees. There are uncertainties in these estimates, due to variations in kink opening angle and numerical errors. Nevertheless, they show that the gravitational power grows with n, and that $\Gamma_k \sim$ 50–100. This can also be understood heuristically in the following way. We associate each left-mover with a right-mover, each pair effectively forming a small smooth loop of size $l \sim n^{-1}$. The large loop, which is of unit length, is "assembled" by n of these small loops. Since the gravitational power of a closed loop depends on its shape but is independent of its size [9], the total power of the very kinky loop will be proportional to n times the power of each small loop, in accord with the above result. This argument suggests that Γ_k is of order the gravitational power Γ of a single smooth closed loop, i.e., $\Gamma_k \sim \Gamma \sim 50$, as is found in Ref. 9 . The factor $\bar{f}$ is a measure of both the dependence of the power of the small loops on their shape and the accuracy of this argument.

Gravitational radiation will dampen the kinks by increasing their opening angle θ. This is because the relevant time-scale for decay of transverse directions is set by the kink oscillations. Longitudinal decay is set by the slower oscillations of the average motion of the long string, and this time is n times longer. (The long string is also prevented from shrinking due to stretching by the expanding universe.) In Ref. 8 , the evolution of a kinky loop is traced, with the gravitational backreaction effects included. The kinks are found to decay much faster than the large loop as a whole, with a time-scale $t \sim l/(\Gamma_k G\mu)$, where again, Γ_k is of order 50. It is easy to see why this is the kink lifetime. The energy per unit proper length carried by n kinks is of order μ. Dividing this by the gravitational power per unit proper length, which we find to be $\Gamma_k G\mu^2\ n$, gives $t_{decay} \sim 1/(\Gamma_k G\mu\ n) = l/(\Gamma_k G\mu)$. The results of Ref. 8 thus support our argument above for the gravitational power of a kinky string, with $\Gamma_k \sim 50$.

Equation (1) has important consequences for the scaling solution. We define the scale length L from the long string density as $\rho_{LS} \equiv \mu/L^2$ [1]. Thus, L^{-2} is the proper length of long strings per unit volume, and we must multiply by this factor to convert quantities per unit proper length to quantities per unit volume. From equation (1) , the long string network loses gravitational energy at a rate $-\dot{\rho}_{grav} = \Gamma_k G\mu\ n\ \rho_{LS}$. Let $\dot{n}_+$ be the rate of increase of kink density due to loop formation, and $\dot{n}_-$ be the rate of decrease of kink density due to decay by gravitational backreaction. Every loop chopped off the long string adds 2 kinks and must increase the linear kink density. Thus, the energy loss from loop formation is directly related to kink production through $-\dot{\rho}_{loop} = c_2(\mu/n)\,\dot{n}_+/L^2 = c_2(\dot{n}_+/n)\ \rho_{LS}$, since the typical loop produced has energy of order μ/n. From above, the decay time-scale for kinks is $\Gamma_k G\mu\ n$, and so the long strings lose kinks at a rate $\dot{n}_-/n = c_1\Gamma_k G\mu\ n$. (Here, c_1, c_2 and c_3 are dimensionless constants of order unity.) Finally, the time-scale for chopping off loops and adding kinks is set by the curvature scale of the long strings, and so $\dot{n}_+/n = c_3/L$. This must be the case, if the scaling solution is to hold for the long strings, and the numerical simulations [1] support this assertion [12],[13].

Combining the above relations, the evolution of the kink density is given by :

$$\frac{\dot{n}}{n} = \frac{c_3}{L} - c_1 \Gamma_k G\mu \, n \quad . \tag{2}$$

Here, we have omitted the term that includes kink decay due to stretching [1], namely $-(\dot{a}/a)(1-2v^2)$, where v^2 is the mean square velocity of the long strings. As Albrecht and Turok explain in Ref. 1 , v^2 is expected, and found to be, very close to $\frac{1}{2}$. Hence the decay of kinks due to stretching is negligible, and we will neglect this effect in our discussion. (Recently, the effects of kink stretching on the string network have been considered [14]; nevertheless, the effects of gravitational radiation will be much larger.)

The kink density n cannot increase indefinitely. Initially, the long string network governs the growth of n. However, the kink decay time decreases as the kink density increases, and at some point becomes comparable to the characteristic kink production time from loop formation. From equation (2) , this occurs when $n^{-1} = l \sim (c_1/c_3)\Gamma_k G\mu\, L$. Afterwards, kink production is effectively choked, and, provided that the scale length L grows no faster than t, l asymptotically grows like L. Thus, equation (2) links the small-scale structure to the characteristic scale of the long string network.

If the long string density does reach scaling, so that L is proportional to the horizon size, then the typical inter-kink distance l will also scale with the horizon. We write $L = 2\gamma t$, substitute into equation (2) , and solve for the kink density as a function of time : $n\,t = N_0(t/t_0)^{1+\frac{c_3}{2\gamma}}\left[1 + \alpha N_0[(t/t_0)^{1+\frac{c_3}{2\gamma}} - 1]\right]^{-1}$, where $N_0 \equiv n_0 t_0$, $\alpha \equiv (c_1\Gamma_k G\mu)/(\frac{c_3}{2\gamma}+1)$, and subscripts refer to initial values. Fig. 1 shows the solution for different values of N_0. Initially, n grows as a power with time, namely $n \sim n_0(t/t_0)^{\frac{c_3}{2\gamma}}$. Note that $c_3/2\gamma < 1$, since the time-scale for adding kinks on the long string must be greater than the Hubble time t. When $n \sim (\alpha t)^{-1}$, then scaling takes over, and the inter-kink distance scales with the horizon : $l = \alpha t$.

Now, the evolution of the long string density is given by

$$\frac{\dot{\rho}_{LS}}{\rho_{LS}} = -2\frac{\dot{a}}{a}(1+v^2) - \frac{c_2 c_3}{L} - \Gamma_k G\mu \, n \quad , \tag{3}$$

where the first term includes the stretching of the long strings in the expanding universe [1]. There is a relation between v^2 and the chopping probability c_3 that ensures scaling [1], i.e, $L \propto t$; however, for simplicity we shall take $v^2 = \frac{1}{2}$. The scaling criterion then becomes : $(\Gamma_k G\mu)\alpha^{-1} = \frac{1}{2} - c_2 \frac{c_1}{2\gamma} < \frac{1}{2}$. Hence, $\alpha > 2\Gamma_k G\mu$. Combining with the result in the previous paragraph, we find that :

$$l = \alpha t = \frac{2(1 + c_1 c_2)}{(1 + 2c_2)} \, (\Gamma_k G\mu) t > 2\Gamma_k G\mu \, t \quad . \tag{4}$$

Here, we have derived an expression for the inter-kink distance that depends solely upon factors that describe the adding and decay of kinks on small scales, namely c_1 and c_2. This expression is independent of factors like c_3 and γ that describe the long string network. We have thus arrived at a "two-scale" scaling solution, with the long string scale length being a sizeable fraction of the horizon, and with gravitational backreaction forcing the kink density to scale with the horizon size, with small-scale structure of size of order, but no smaller than, $\Gamma_k G\mu$ times the horizon, where $\Gamma_k \sim 50$.

The loops that are chopped off from this network scale in the same fashion as the small-scale structure on the long strings. This means that loops are a tiny fraction of the horizon size, and thus the loop-seeded scenario of galaxy formation is most likely ruled out. The gravitational radiation from the long strings is dominated by the contribution from small-scales, from equation (1), and it will be very similar, in both frequency and amplitude, to the gravitational radiation from the loops. This is because, in the new scenario [3], the total energy in long strings and in small loops is about the same. Thus the limits on the string parameter $G\mu$ from millisecond pulsar measurements are not altered significantly when the radiation from long strings is included.

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FIGURE CAPTIONS

1) The product $n\,t$ as a function of the ratio t/t_0, as given by equation (2), for different values of $N_0 \equiv n_0 t_0$. Here, $c_3/2\gamma = 0.5$, and $c_1 \Gamma_k G\mu = 10^{-4}$.

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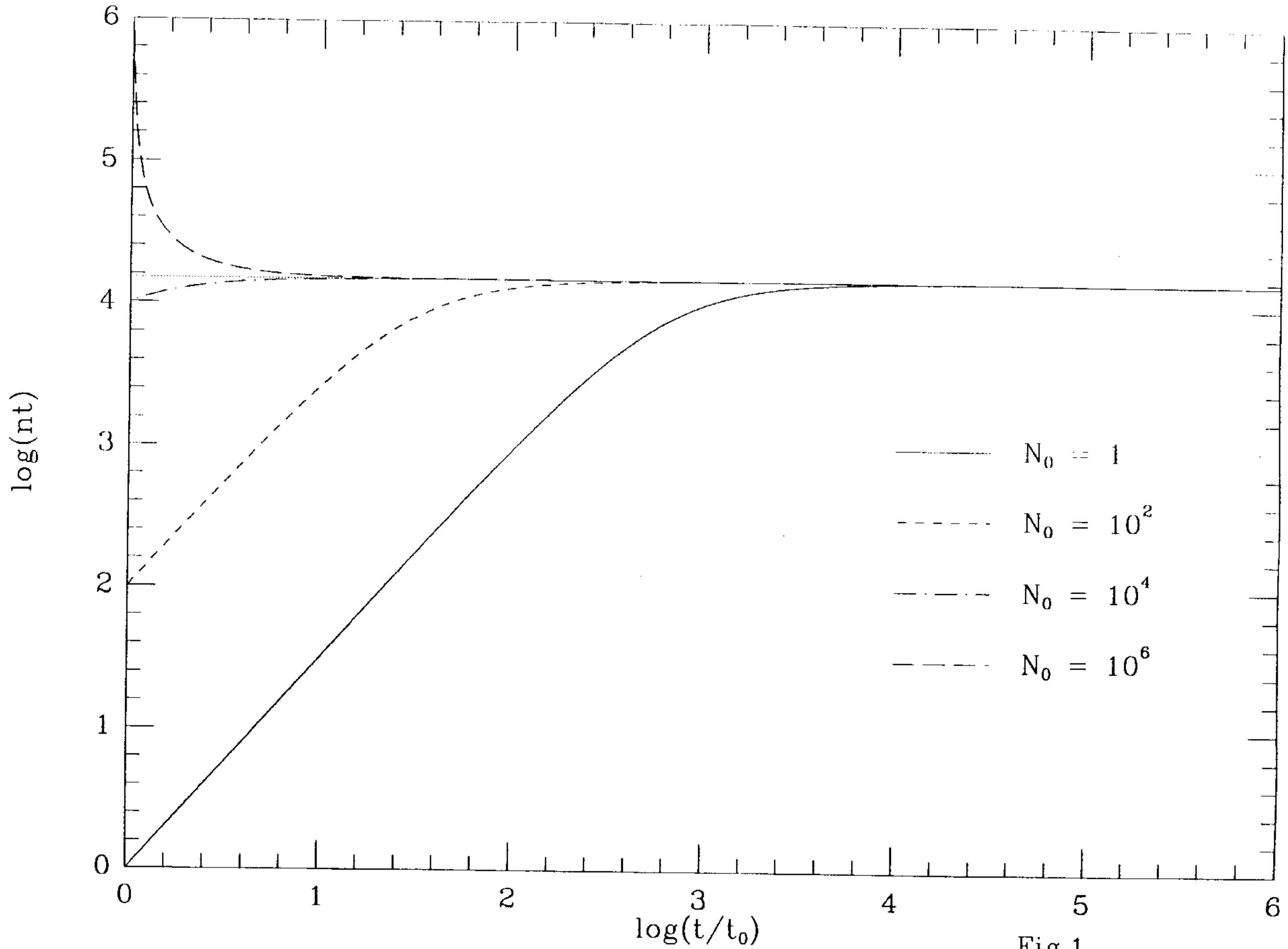

Fig.1